Statistics of One-Way Internet Packet Delays

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The Data

- Packet transmission between two CQOS C-nodes (a vector).
- Internet transmission (not a dedicated link)
- Packets transmitted periodically over 300 second periods
- Each 300 second period $\rightarrow$ 'Measurement Record'
- A dataset consists of many sequential Measurement Records on a particular vector
- Three datasets: # 1, # 3 and # 4

Measurement Record (300 seconds)

- M packets with fixed length of 576 bytes dispatched periodically
- GPS synchronized send (t_s) and receive (t_r) times of each packet measured/recorded
- One-way delay $d = t_r - t_s$
- The total number of sent (M) and received ($\leq M$) packets recorded
- Duplicated and dropped packets recorded

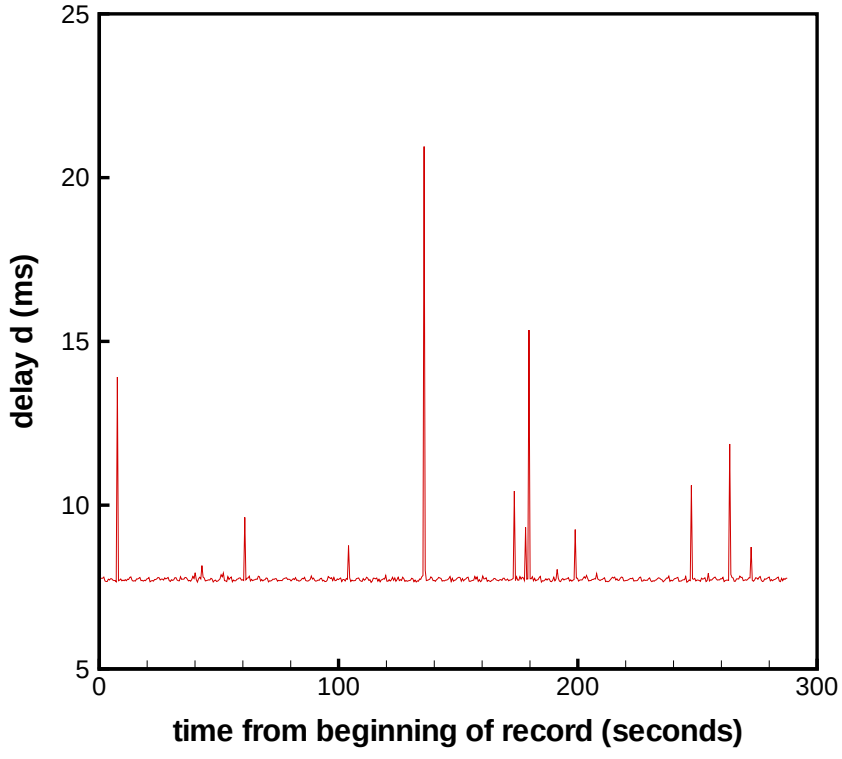
Dataset parameters

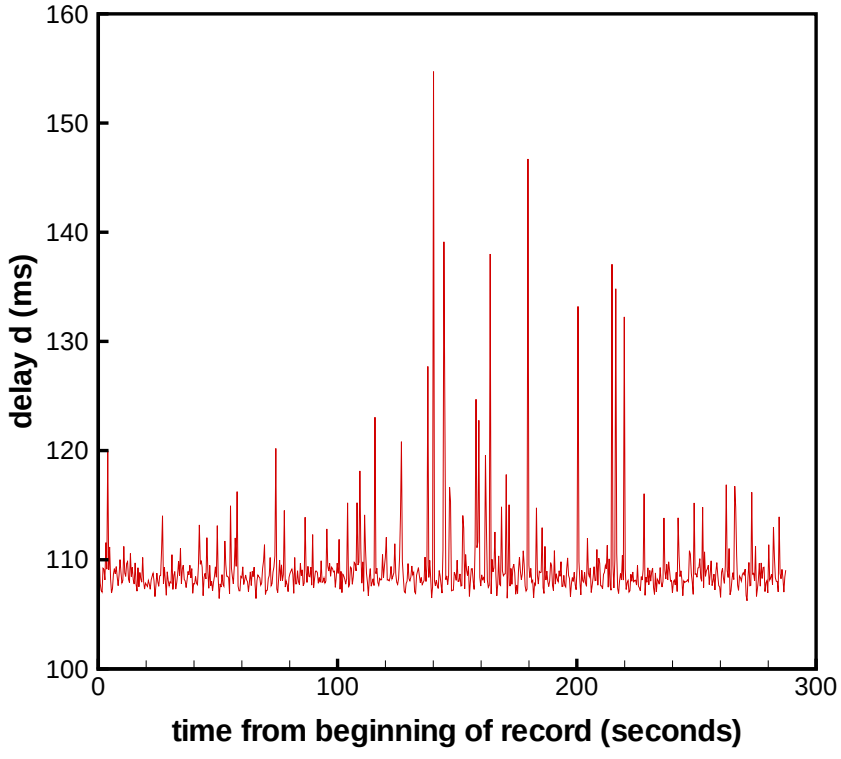
Dataset	Number of records	Measurement period (days)	Pkts/300 secs (M).	Occupancy fraction ν	Vector (Hop Count)
# 1	621	2.2	611	0.0063	Local (11)
# 3	2033	7.1	9556	0.092	Local (11)
# 4	1017	3.5	730	0.0073	London (22)

Parameters defining datasets # 1, # 3 and # 4. Bandwidith $= 1.5 \times 10^6$ bps, packet length $= 576$ bytes, utilization $\rho = 1.0$.

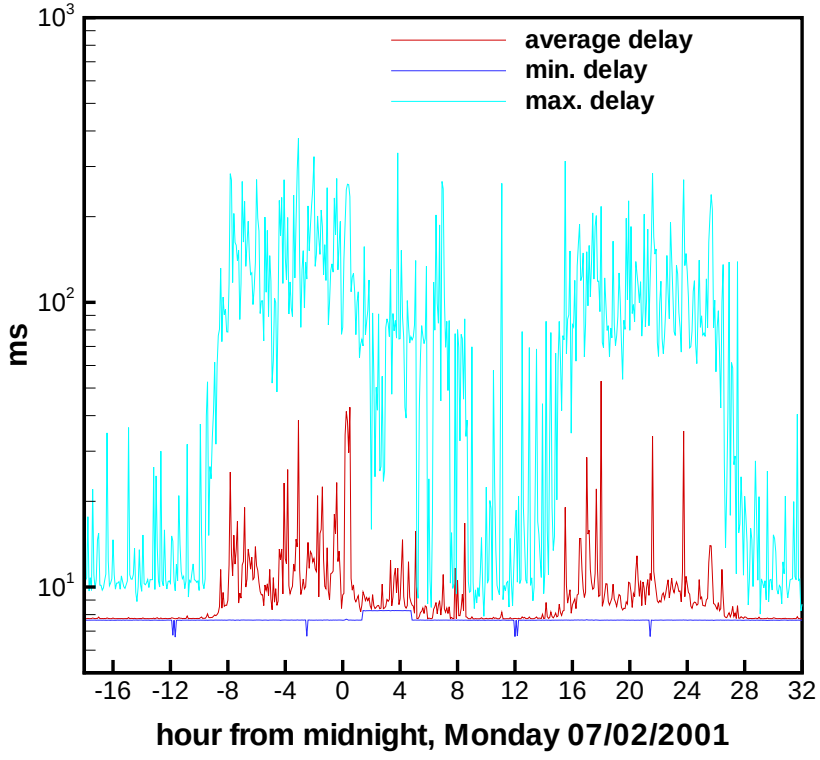
Statistics (300 second measurement period)

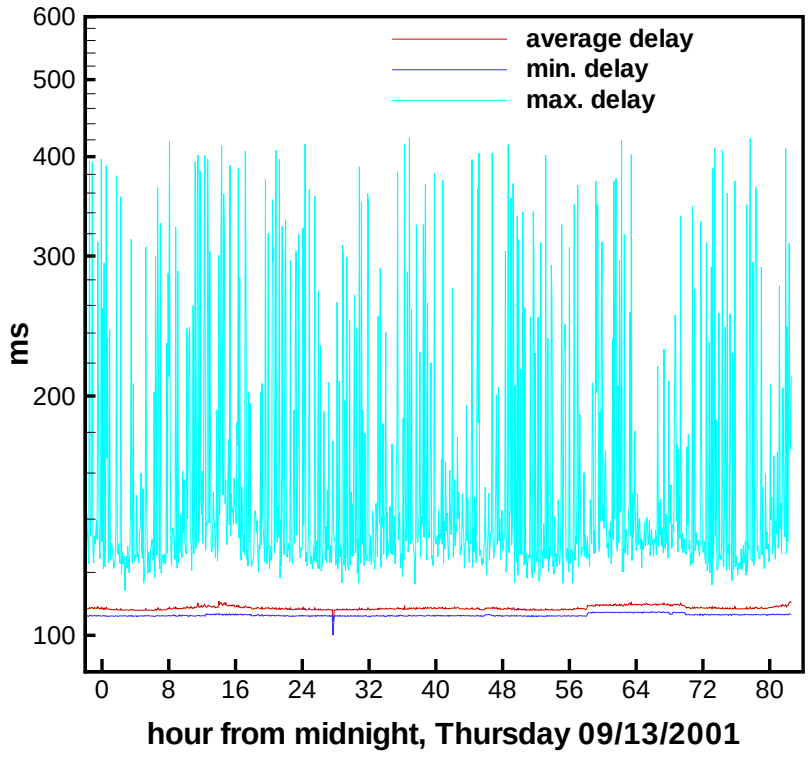
- Mean delay: $< d >= \frac{1}{M_r} \sum_{i=1}^{M_r} d_i$
- Standard deviation: $s^2 = \frac{1}{M_r - 1} \sum_{i=1}^{M_r} (d_i - < d >)^2$
- Minimum and maximum delay d_{min}, d_{max}
- Probability density (pdf). Autocorrelations. Power spectra.



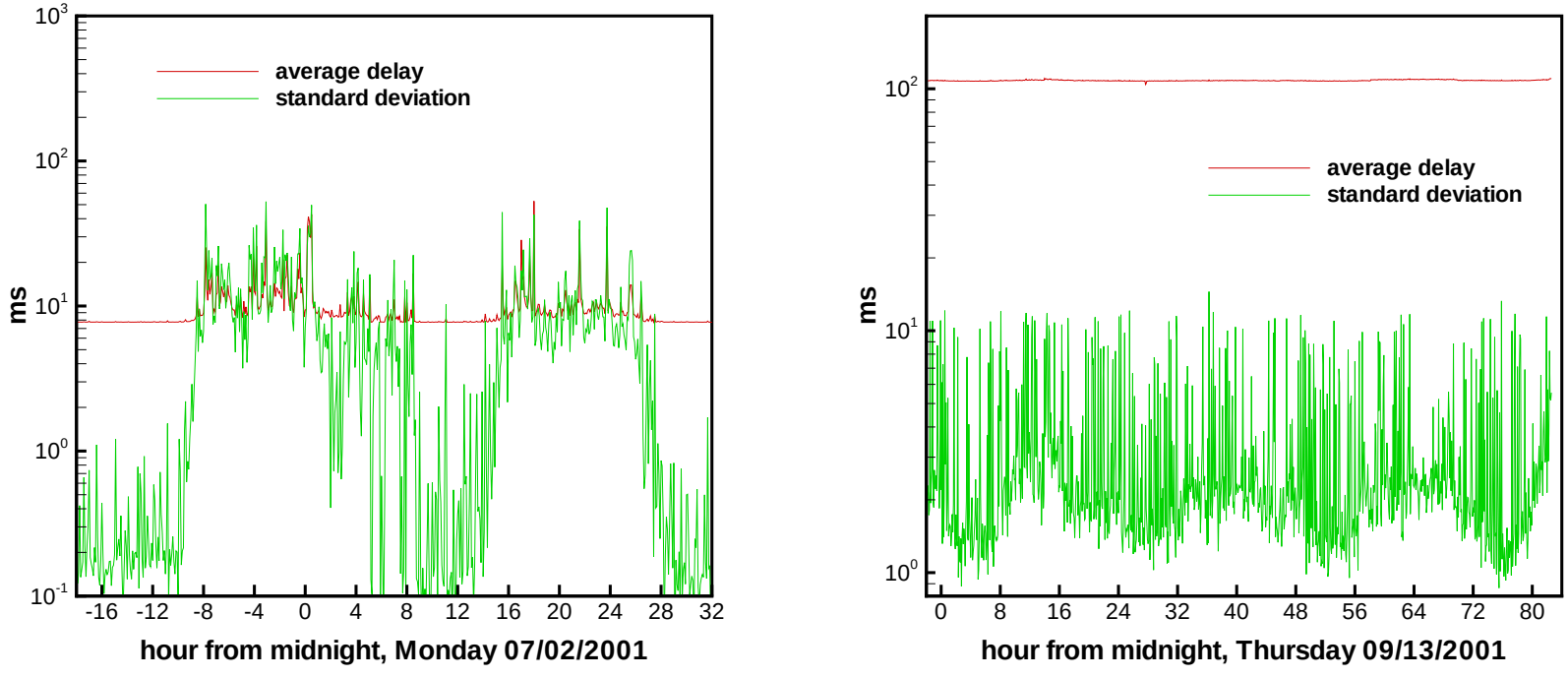


Typical time series of delay over a 300 second measurement record. Left; dataset # 1. Right; dataset # 4.

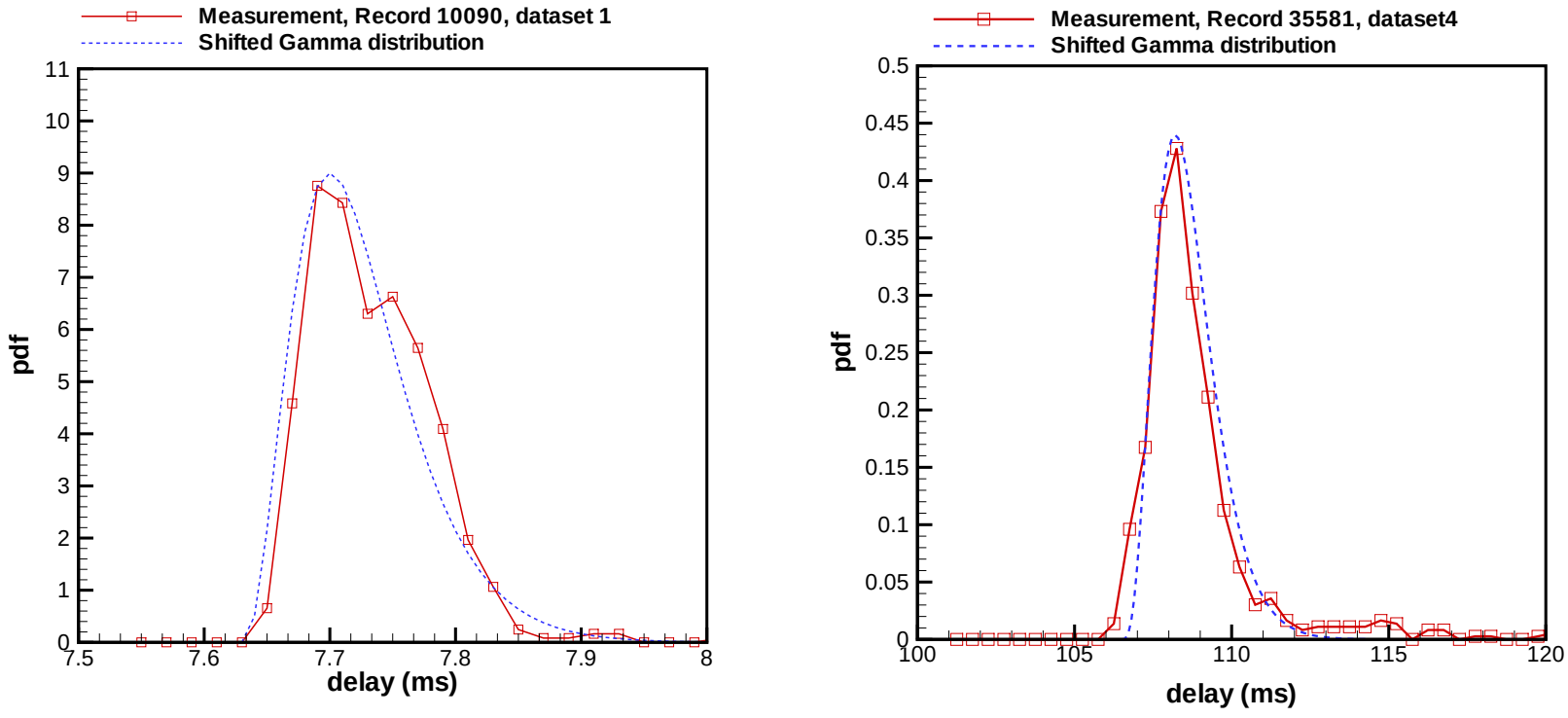




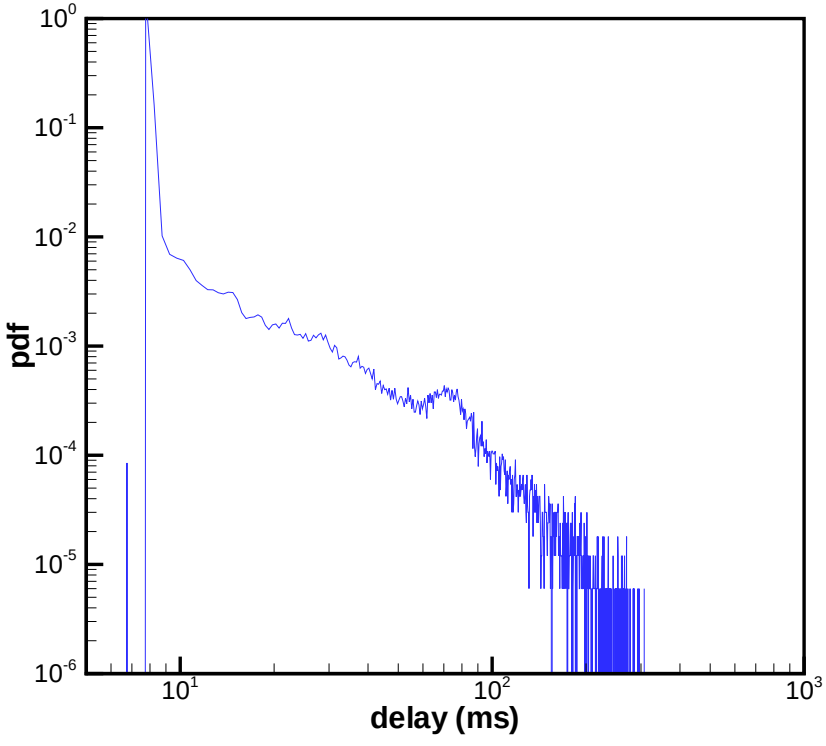
Average, minimum and maximum delay over consecutive 300 second measurement records. Left; dataset # 1. Right; dataset # 4.

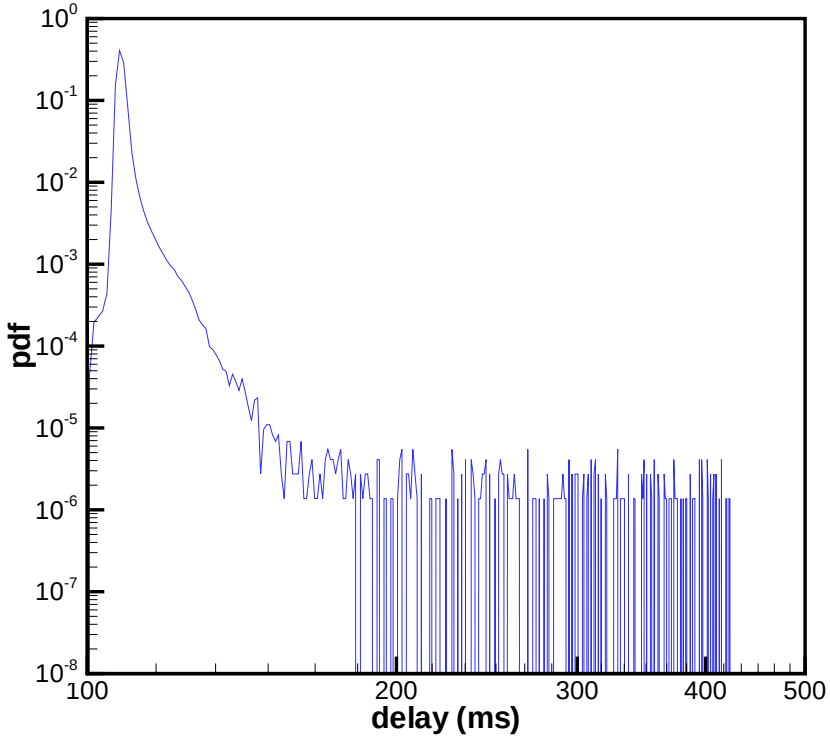


Average delay & standard deviation over consecutive 300 second measurement records. Left; dataset # 1. Right; dataset # 4.



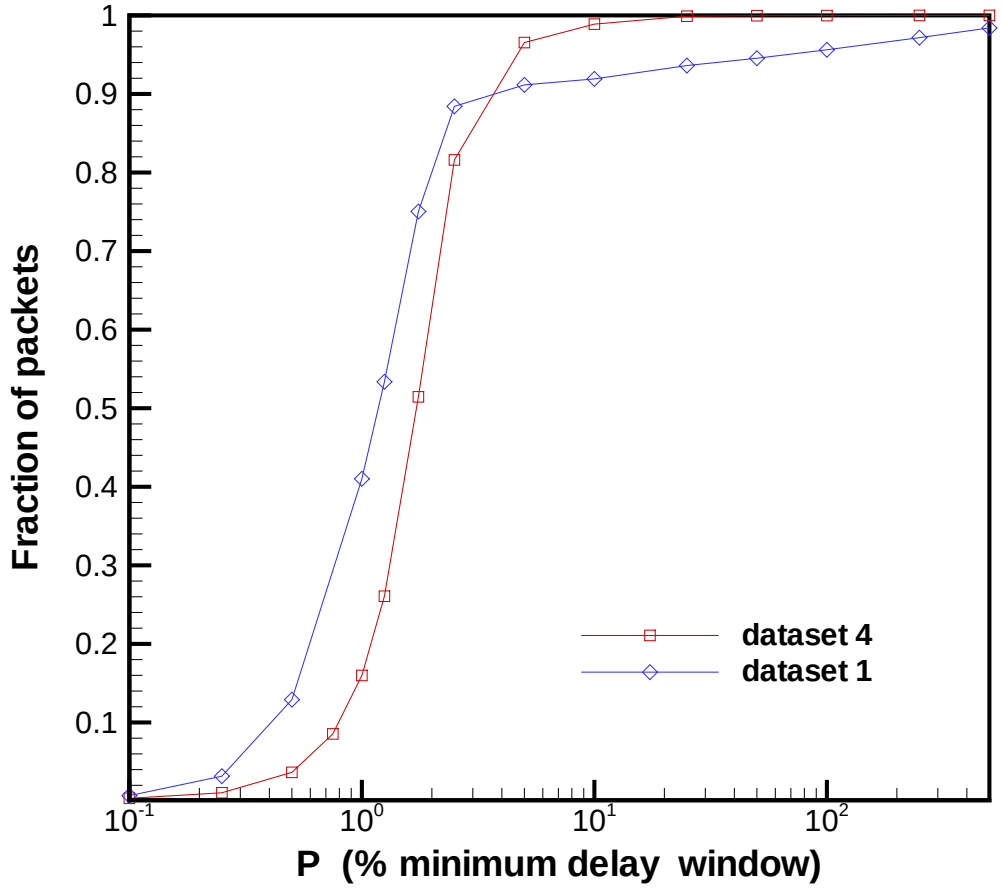
Delay pdfs over typical 300 second measurement record. Left; dataset # 1. Right; dataset # 4. Approximated by shifted Gamma distribution [Mukherjee (1992)].



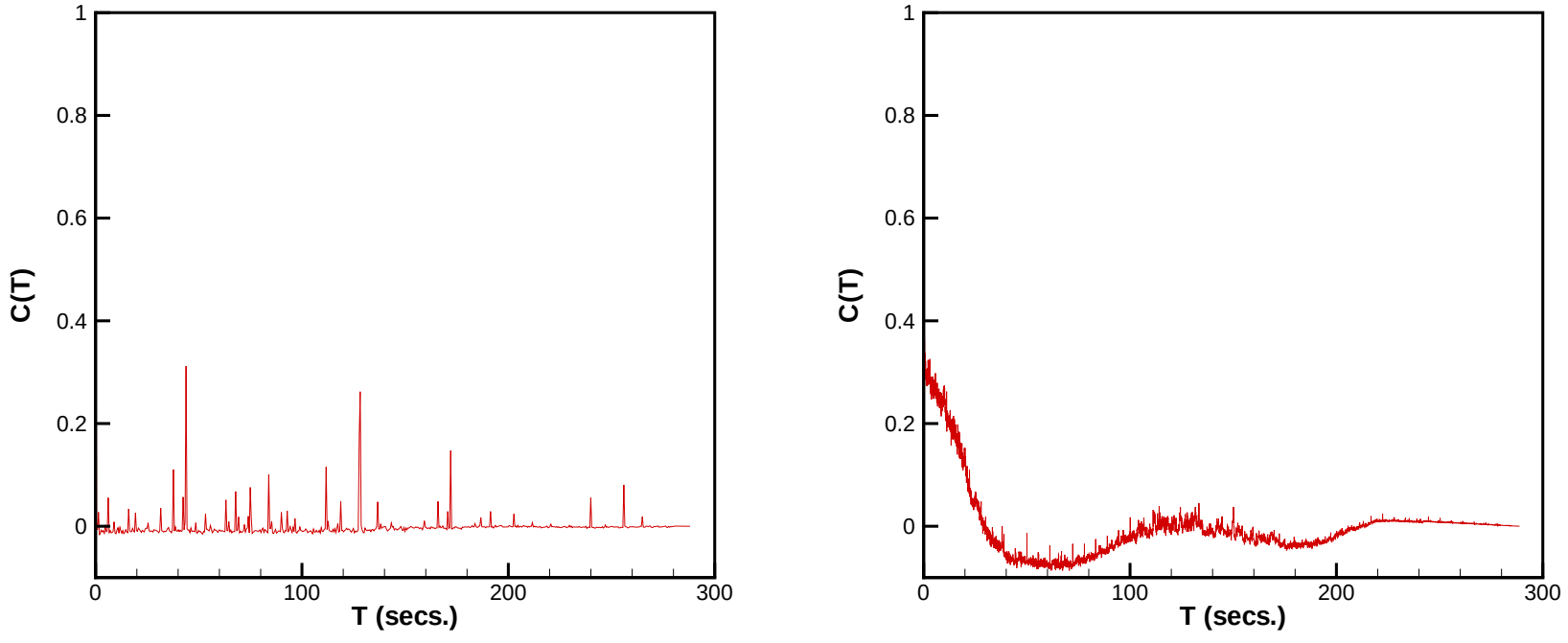


Delay pdfs over whole dataset.

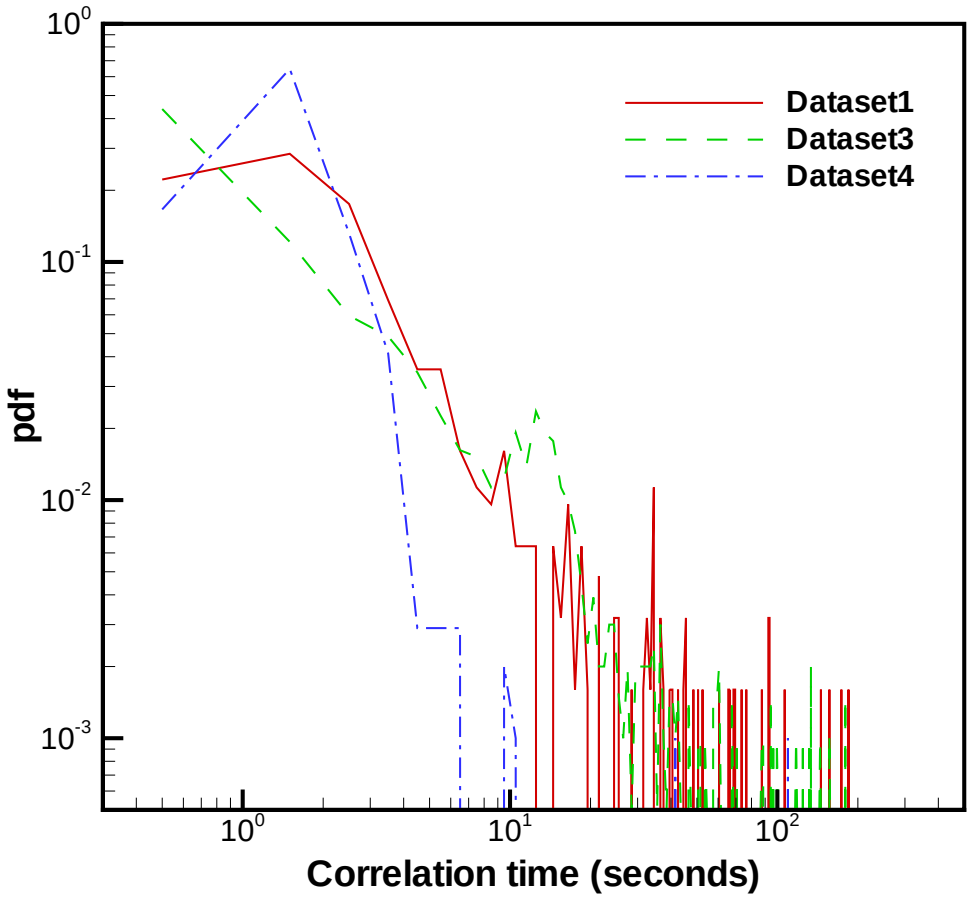
Left; dataset # 1. Right; dataset # 4.



Average fraction of packets in measurement record with delay within $P\%$ minimum delay window for datasets #1 and # 4



Autocorrelation function over one typical 300 second measurement record. Left; dataset # 1. Right; dataset # 3.



Pdf of delay autocorrelation time over typical 300 second measurement records. Three datasets.

Power spectrum of delay

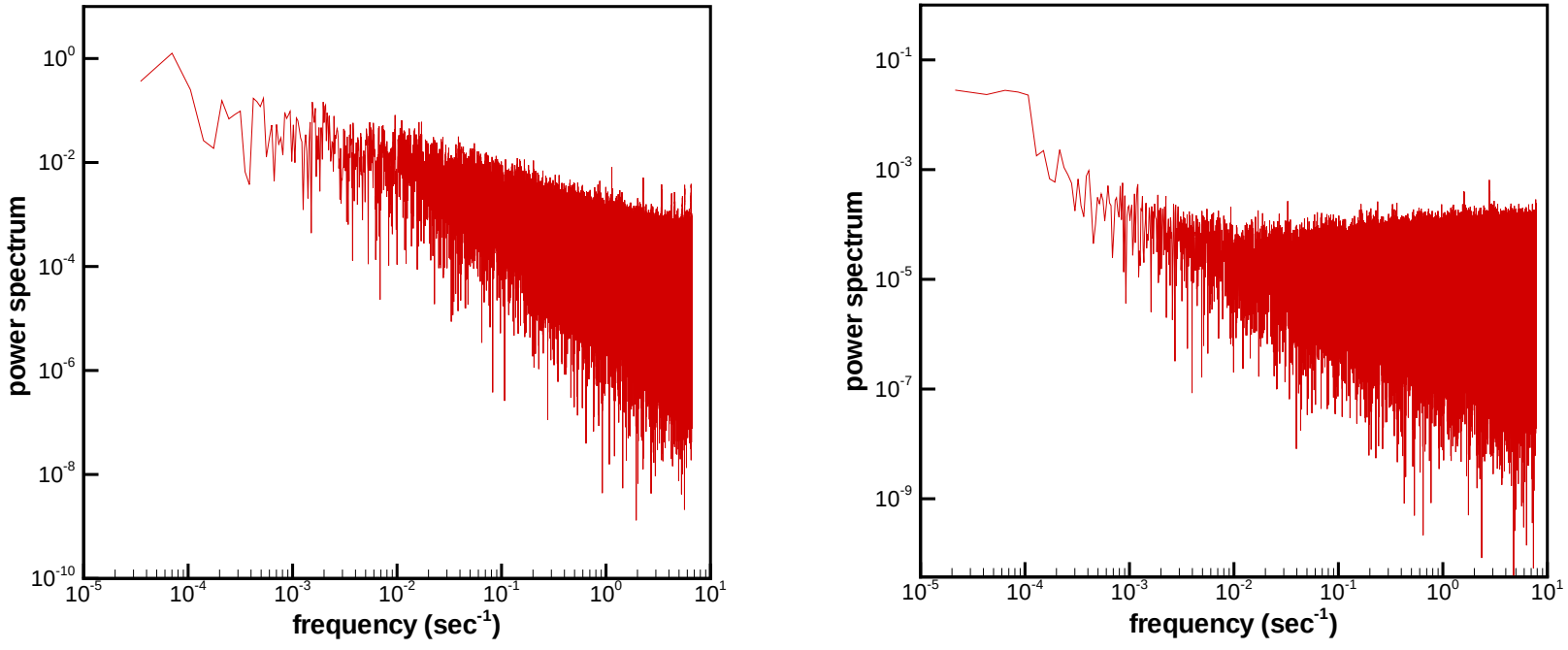
- Delay time series consists of concatenated records

- Dataset # 1; 380,030 packets.
  Dataset # 4; 729,844 packets

- Fourier series for delay time series

$$d(t) = \sum_{k=-N/2}^{N/2-1} \hat{d}_k \mathrm{e}^{i\omega_k t}, \qquad , \omega_k = \frac{2\,\pi\,k}{T}, \qquad d_{-k} = d_k^*,$$

- Power spectrum $= |\hat{d}_k(\omega)|^2$



Power spectrum of measured delay time series.
Left, dataset # 1. Right, dataset # 4

Summary; statistics of one-way delay

- Pdf of one-way delay well approximated by shifted-Gamma distributions.
- Data subject to varying degrees of non-stationarity
- Mean delay time series show:
  - ▷ strong temporal variability in local datasets
  - ▷ lesser temporal variability in international dataset
- Autocorrelation time for delay series $\sim 2 - 10$ seconds
- Power spectra show only expected dominant frequencies